Congruent Triangles

Lesson Synopsis:

In this lesson, the concept of congruence is extended to triangles. Congruent triangles are explored both from the standpoint of congruence transformations and an axiomatic structure. Students justify congruent triangles with congruence transformations and prove triangles congruent with postulates and theorems. Students extend the concepts of congruent triangles to prove further characteristics of triangles using corresponding parts of congruent triangles, CPCTC.

TEKS:

G. 2	***Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures.***
G.2B	**Make conjectures about angles, lines, polygons, ~~circles, and three-dimensional figures~~ and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.**
G.3	***Geometric structure. The student applies logical reasoning to justify and prove mathematical statements.***
G.3B	**Construct and justify statements about geometric figures and their properties.**
G.3C	**Use logical reasoning to prove statements are true and find counter examples to disprove statements that are false.**
G.7	***Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.***
G.7A	**Use ~~one- and~~ two-dimensional coordinate systems to represent ~~points, lines, rays, line segments, and~~ figures.**
G.10	***Congruence and the geometry of size. The student applies the concept of congruence to justify properties of figures and solve problems.***
G.10A	**Use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane.**
G.10B	**Justify and apply triangle congruence relationships.**

GETTING READY FOR INSTRUCTION

Performance Indicator(s):

- Identify congruent transformations and use them to make and justify conjectures and to solve problems. (G.3B; G.10A)
  ELPS ELPS: 1E, 2E, 2I, 3J, 4F, 5G
- Apply triangle congruency postulates and theorems to justify that two triangles are congruent. (G.2B; G.3B, G.3C; G.7A; G.10A, G.10B)
  ELPS ELPS: 1E, 2E, 2I, 3K, 4F, 5G
- Determine if triangles are congruent by applying congruence relationships and apply CPCTC to justify the congruence of parts of triangles. (G.2B; G.3C; G.10B)
  ELPS ELPS: 1E, 2E, 2I, 3K, 4F, 5G

Key Understandings and Guiding Questions:

- Translations (slides), rotations, or reflections (flips) are congruent transformations because they maintain congruence of shape and size.
  — What does the term “congruent transformations” mean?
  — Does the dilation of a triangle maintain congruency?
- Congruent transformations can be used to justify conjectures.
  — When writing a statement to show that two triangles are congruent, how must the letters representing the vertices be arranged?

- Congruent triangles have specific characteristics, including the fact that corresponding parts of congruent triangles are congruent.
  - — What does the term “corresponding parts of polygons” mean?
  - — How are corresponding angles of congruent triangles related?
  - — How are corresponding sides of congruent triangles related?
- Characteristics of congruent triangles can be used to prove triangles are congruent.
  - — What triangle congruence relationships must be true to prove triangles congruent?
  - — How can coordinate geometry be used to prove triangles congruent?
  - — How can triangles be proven congruent through specific postulates and theorems?
  - — What postulates and theorems allow triangles to be proven congruent?
- Congruent triangle proofs can be extended to prove congruence of parts.
  - — What is CPCTC and why is it important?

Vocabulary of Instruction

- congruent transformations
- congruent triangles
- CPCTC
- SSS
- SAS
- ASA
- AAS
- HL
- HA
- LL

Materials:

- graphing calculator
- sandwich name cards
- card stock (construction paper) of six different colors
- ruler
- scissors
- compass
- protractor
- tape or glue stick
- markers
- colored pencils
- chart paper
- chart markers

Resources:

- **STATE RESOURCES:**
  - — **Mathematics TEKS Toolkit:** Clarifying Activity/Lesson,/Assessments
    http://www.utdanacenter.org/mathtoolkit/index.php
  - — **TEXTEAMS: Geometry for All Institute:** II – Transformationally Speaking with Reflection; Act. 3 (Fast Food Factory)

Advance Preparation:

1. Handout: **Congruence Transformations and Triangles** (1 per student)
2. Cards: **Sandwich Name Cards** (1 set per teacher, run off on card stock, laminate, cut)
3. Handout: **Savory Sandwich Activity** (1 per pair)
4. Handout: **The Savory Sandwich Menu** (1 per pair)
5. Handout: **Congruent Triangle Theorems** (1 per student)
6. Handout: **What Makes Triangles Congruent?** (1 per student)
7. Handout: **Proving Triangles Congruent and CPCTC** (1 per student)
8. Handout: **Sneaky Triangles** (1 per student)
9. Handout: **Tall Tom’s Short Pants** (1 per student)
10. Handout: **Over the Roof** (1 per student)
11. Handout: **Evaluating Congruent Triangles** (1 per student)

Background Information:

In this lesson, students continue exploring congruence relationships from the standpoint of triangles. Students prove and justify congruent triangle relationships using a variety of methods including congruence transformations, and postulates and theorems.

GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

Instructors are encouraged to supplement, differentiate and substitute resources, materials, and activities to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this Planning document is located at www.cscope.us/sup_plan_temp.doc. If a supplement is created electronically,

users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource for future reference.

INSTRUCTIONAL PROCEDURES

Instructional Procedures

ENGAGE

1. Distribute the handout: **Congruence Transformations and Triangles** to each student.
2. Students should work on the activity individually.
3. When students have completed the activity, they should get in groups of 3 to 4 and compare results, making corrections as needed.
4. Lead the class in a whole group discussion with the following questions.
   - **What transformation creates a reflection in the y-axis?** *Answers will vary. Sample: Taking the opposite of the x-coordinate.*
   - **What transformation creates a rotation of 180°?** *Answers will vary. Sample: Taking the opposite of both the x-coordinate and y-coordinate.*
   - **What transformation creates a horizontal translation?** *Answers will vary. Sample: Adding or subtracting an amount from the x-coordinate.*
   - **What transformation creates a vertical translation?** *Answers will vary. Sample: Adding or subtracting an amount from the y-coordinate.*
   - **What are the congruent transformations?** *Translations, reflections, rotations.*
   - **Why are they called congruent transformations?** *They maintain size and shape of the original image.*
   - **How are dilations different from the congruent transformations?** *Dilations maintain shape but not size, so the new image is not congruent to the original image.*

Notes for Teacher

NOTE: 1 Day = 50 minutes
Suggested Day 1
MATERIALS
- Handout: **Congruence Transformations and Triangles** (1 per student)
- graphing calculator

TEACHER NOTE
The purpose of this activity is to use the graphing calculator and congruence transformations to illustrate congruent triangle mappings.

STATE RESOURCES
Mathematics TEKS Toolkit: Clarifying Activity/Lesson/Assessment may be used to reinforce these concepts or used as alternate activities.

EXPLORE 1

1. Before class, post the **Sandwich Name Cards** on the board. Students will tape the “sandwiches” they make under the appropriate name cards.
2. Group in pairs. Make sure each pair has the necessary materials, including six colors card stock, ruler, scissors, compass, protractor, tape or glue stick, markers, and colored pencils.
3. Distribute the handout: **Savory Sandwich Activity** to each pair of students. Go over the procedure for the activity with students.
4. Distribute the handout: **The Savory Sandwich Menu** to each pair of students and have students study the Chef’s Instructions and cut out six sandwiches according to the instructions.
5. Students should label each vertex of the triangles appropriately. Be sure to label <u>inside</u> the triangle!
6. Students should label all sandwiches with the sandwich name and initials.
7. Students should label all sandwiches with the angle and side measurements.
8. Have each pair tape their sandwiches under the representative Sandwich Name Page.
9. When all sandwiches have been put up, compare and determine which sandwiches are always the same in size and shape and which are not!
10. Have students correct or complete the Conclusions for the activity. This can be completed as homework, if necessary.

As students begin to post their triangles to the sandwich name pages, pay particular attention to #3 Sensibly Satisfying Artichoke (side-side-angle). There are two different ways to construct this triangle!

Suggested Day 2
MATERIALS
- Cards: **Sandwich Name Cards** (1 set per teacher, run off on card stock, laminate, cut)
- Handout: **Savory Sandwich Activity** (1 per pair)
- Handout: **The Savory Sandwich Menu** (1 per pair)
- card stock (construction paper) of six different colors
- ruler
- scissors
- compass
- protractor
- tape or glue stick
- markers
- colored pencils

TEACHER NOTE
In this activity, students will use concrete models to discover which congruence relationships between triangle parts determine if the triangles are congruent.

Instructional Procedures

Notes for Teacher

TEACHER NOTE
Sometimes in small classes of students, the two different ways to construct a triangle using SSA (Sensibly Satisfying Artichoke) do not always show up. Be prepared to facilitate at least one pair of students to ensure that it does, or have two different SSA triangles constructed ahead of time!

STATE RESOURCES
TEXTEAMS: Geometry for All Institute: II – Transformationally Speaking with Reflection; Act. 3 (Fast Food Factory) may be used to reinforce these concepts or used as alternate activities.

EXPLAIN 1

Day 3
1. Debrief the handout: **Savory Sandwich Activity** in whole-group discussion.
   Facilitation Questions
   - **Which of the sandwiches (methods of triangle construction) resulted in congruent triangles?** *SSS, SAS, ASA, AAS*
   - **Which of the sandwiches (methods of triangle construction) resulted in triangles that are not congruent? Why do you think this is so? Explain.** *AAA, SSA*
   - **What does this tell you about the ways to show triangles are congruent?** *Answers will vary. Sample: SSS, SAS, ASA, AAS*
2. Distribute the handout: **Congruent Triangle Theorems** to each student. Go over the postulates and theorems in whole class instruction.

Day 4
3. Use Examples 1-5 to help clarify students' understanding.
4. Distribute the handout: **What Makes Triangles Congruent?** to each student.
5. Have students complete handout: **What Makes Triangles Congruent?** in order to demonstrate their understanding. This can be assigned as homework, if necessary.

Suggested Days 3-4
MATERIALS:
- Handout: **Congruent Triangle Theorems** (1 per student)
- Handout: **What Makes Triangles Congruent?** (1 per student)

TEACHER NOTE
Students will investigate formal postulates and theorems that can be used to prove triangles congruent.

TEACHER NOTE
Question #4 on handout: **Congruent Triangle Theorems** involves a flow chart proof. Since students have not completed this type of proof prior to this activity, you may need to facilitate this proof in whole-group instruction.

TEACHER NOTE
In this activity, theorems related to right triangles that are not covered by the handout: **Savory Sandwich Activity** are also investigated. These are LL (Leg-Leg), HL (Hypotenuse-Leg) and HA (Hypotenuse-Angle). However, these are actually special cases of the other theorems. For example, LL is actually SAS; HA is actually AAS; and HL is actually the one case for which SSA works.

EXPLORE/EXPLAIN 2

1. Debrief handout: **What Makes Triangles Congruent?** by having students get with a partner and compare answers. Have volunteer pairs share results in whole group.

Suggested Day 5
MATERIALS
- Handout: **Proving Triangles Congruent and CPCTC** (1 per

Instructional Procedures

2. Distribute the handout: **Proving Triangles Congruent and CPCTC** to each student.
3. Go over notes and examples in whole-group instruction.
4. Assign Practice Problems to be worked individually or in pairs. Monitor students to check for understanding.

Notes for Teacher

student)

TEACHER NOTE
In this activity, students will extend congruent triangle proofs to prove corresponding parts of congruent triangles are congruent.

SUPPLEMENTARY MATERIALS
- Handout: **Sneaky Triangles** (1 per student)
- Handout: **Tall Tom's Short Pants** (1 per student)

Supplementary materials can be used for additional practice in corresponding parts and congruent triangles as needed.

ELABORATE

1. Go over the Practice Problems in whole-group discussion.
2. Distribute the handout: **Over the Roof** to each student.
3. Have students work in small groups to complete the handout.
4. When finished, have students record results on chart paper and post.
5. Debrief the activity by having groups explain their results on the display charts.

Suggested Day 6
MATERIALS
- Handout: **Over the Roof** (1 per student)
- chart paper
- chart markers

TEACHER NOTE
In this activity, students will apply triangle relationships, congruent transformations, and congruent triangle postulates and theorems to construction of roof trusses.

EVALUATE

1. Distribute the handout: **Evaluating Congruent Triangles** to each student.
2. Students should work the handout individually as an assessment.

Suggested Day 7
MATERIALS
- Handout: **Evaluating Congruent Triangles** (1 per student)

TEACHER NOTE
This activity should be completed independently to assess student knowledge of the concepts taught in the lesson.

TAKS CONNECTION
Grade 11 TAKS 2004 #10
Grade 11 July TAKS 2004 #50

Grade 11 July TAKS 2006 #7

Congruence Transformations and Triangles KEY

Procedure for demonstration using graphing calculator:

1. *Important!!!* Before entering any data, choose the standard window (ZOOM, 6:ZStandard), and then the square window (ZOOM, 5:ZSquare). This sets a reasonable viewing window and squares the aspect ratio so that figures are not distorted.

2. Using the list editor feature of the calculator enter the following lists.

L1	L2	L3	L4	L5
-2	3	2	-3	6
-6	4	6	-4	2
-9	9	9	-9	-1
-2	3	2	-3	6

3. Using the StatPlot feature of your calculator, create a connected plot of L1 vs. L2. The result is a triangle in quadrant 2.

4. To reflect the triangle, create a second connected plot of L3 vs. L2.
   a. What is the resulting transformation? The result is a reflection over the y-axis.
   b. What appears to be true about the image and the pre-image? The triangles are congruent.
   c. Is your answer to part b consistent with your earlier findings about reflections? Yes

5. To rotate the original figure, modify the second connected plot so that it is L3 vs. L4.
   a. What is the resulting transformation? The result is a rotation of 180^o about the origin.
   b. What appears to be true about the image and the pre-image? The triangles are congruent.
   c. Is your answer to part b consistent with your earlier findings about rotations? Yes

6. To translate the original figure, modify the second connected plot so that it is L5 vs. L2.
   a. What is the resulting transformation? The result is a translation of 8 units to the right.
   b. What appears to be true about the image and the pre-image? The triangles are congruent.
   c. Is your answer to part b consistent with your earlier findings about translations? Yes

7. Create two additional lists (or alter L3 and L4) by multiplying L1 and L2 by the *same* scale factor. (If using a scale factor larger than 1.3, you will have to alter the viewing window.) Create a second connected plot along with the original.
   a. What is the resulting transformation? The result is a dilation of the triangle which affects the size. If the scale factor is less than one, it will be smaller. If the scale factor is greater than one, it will be larger.
   b. What appears to be true about the image and the pre-image? The shape remains the same, but the size is either larger or smaller.
   c. Is your answer to part b consistent with your earlier findings about dilations? Yes

8. Which transformations are congruent transformations? Translations, reflections, rotations

Congruence Transformations and Triangles

Procedure for demonstration using graphing calculator:

1. *Important!!!* Before entering any data, choose the standard window (ZOOM, 6:ZStandard), and then the square window (ZOOM, 5:ZSquare). This sets a reasonable viewing window and squares the aspect ratio so that figures are not distorted.

2. Using the list editor feature of the calculator enter the following lists.

L1	L2	L3	L4	L5
-2	3	2	-3	6
-6	4	6	-4	2
-9	9	9	-9	-1
-2	3	2	-3	6

3. Using the StatPlot feature of your calculator, create a connected plot of L1 vs. L2. The result is a triangle in quadrant 2.

4. To reflect the triangle, create a second connected plot of L3 vs. L2.
   a. What is the resulting transformation?
   b. What appears to be true about the image and the pre-image?
   c. Is your answer to part b consistent with your earlier findings about reflections?

5. To rotate the original figure, modify the second connected plot so that it is L3 vs. L4.
   a. What is the resulting transformation?
   b. What appears to be true about the image and the pre-image?
   c. Is your answer to part b consistent with your earlier findings about rotations?

6. To translate the original figure, modify the second connected plot so that it is L5 vs. L2.
   a. What is the resulting transformation?
   b. What appears to be true about the image and the pre-image?
   c. Is your answer to part b consistent with your earlier findings about translations?

7. Create two additional lists (or alter L3 and L4) by multiplying L1 and L2 by the *same* scale factor. (If using a scale factor larger than 1.3, you will have to alter the viewing window.) Create a second connected plot along with the original.
   a. What is the resulting transformation?
   b. What appears to be true about the image and the pre-image?
   c. Is your answer to part b consistent with your earlier findings about dilations?

8. Which transformations are congruent transformations?

Sandwich Name Cards (pp. 1 of 6)

#1 SSS

Sassy Sesame Steak

Sandwich Name Cards (pp. 2 of 6)

#2 AAA

Absolutely Amazing Alligator

Sandwich Name Cards (pp. 3 of 6)

#3 SSA

Sensibly Satisfying Artichoke

Sandwich Name Cards (pp. 4 of 6)

#4 AAS

Aunt Angelina's Squirrel

Sandwich Name Cards (pp. 5 of 6)

#5 SAS

Super Audacious
Swine

Sandwich Name Cards (pp. 6 of 6)

#6 ASA

Astounding Succulent Angus

Savory Sandwich Activity

Materials:

Sandwich Name Cards, card stock (construction paper) of six different colors, ruler, scissors, compass, protractor, tape or glue stick, markers, colored pencils, activity sheet

Procedure:

1. Group in pairs. Get six colors of paper and an activity sheet.
2. Study the Chef's Instructions and cut out six sandwiches according to the instructions.
3. Label each vertex of the triangles appropriately. Be sure to label inside the triangle!
4. Label all sandwiches with their name and initials.
5. Label all sandwiches with the angle and side measurements.
6. Tape the sandwiches under the representative Sandwich Name Page.
7. When all sandwiches have been put up, compare and determine which sandwiches are always the same in size and shape and which are not!

Conclusions:

Convince your boss at The Savory Sandwich why each of the six menu choices may or may not be used to prove triangles congruent.

#1 SSS

#2 AAA

#3 SSA

#4 AAS

#5 SAS

#6 ASA

THE SAVORY SANDWICH MENU

Menu
Sandwiches Extraordinaire

#1 SSS Sassy Sesame Steak
#2 AAA Absolutely Amazing Alligator
#3 SSA Sensibly Satisfying Artichoke
#4 AAS Aunt Alice's Squirrel
#5 SAS Super Audacious Swine
#6 ASA Astounding Succulent Angus

You have been hired at The Savory Sandwich. The chef will test whether you will be allowed to work in the kitchen as an assistant by having you build six extraordinary sandwiches according to the specifications below. If you fail, you will be put in charge of sweeping and mopping floors.

Chef's Instructions

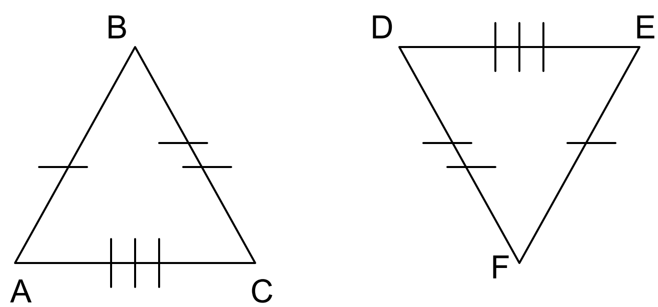
Sandwich	Triangle	Specifications
#1 SSS	ΔABC	AB = 12 in BC = 10 in AC = 8 in
#2 AAA	ΔDEF	$m\angle D = 60$ $m\angle E = 80$ $m\angle F = 40$
#3 SSA	ΔJKL	JK = 5 in JL = 7 in $m\angle K = 40$
#4 AAS	ΔPQR	$m\angle P = 40$ $m\angle R = 60$ PQ = 8 in
#5 SAS	ΔGHI	GH = 8 in $m\angle G = 55$ GI = 6 in
#6 ASA	ΔMNO	$m\angle M = 60$ MN = 8 in $m\angle N = 45$

Be sure to label inside the triangle!!!!

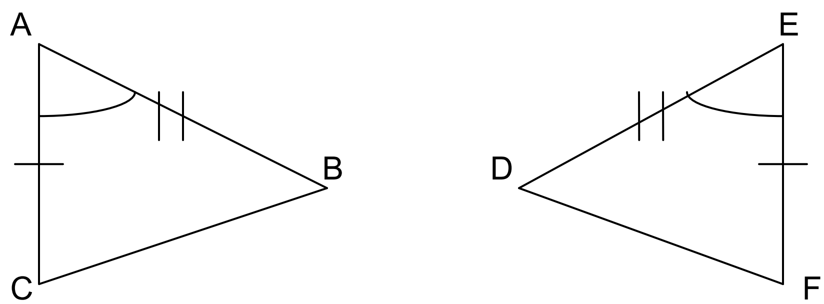
Congruent Triangle Theorems (pp. 1 of 5) KEY

The following are seven methods by which to prove triangles are congruent, without having to find all corresponding parts congruent: SSS, SAS, ASA, AAS, and for right triangles only LL, HA, and HL.

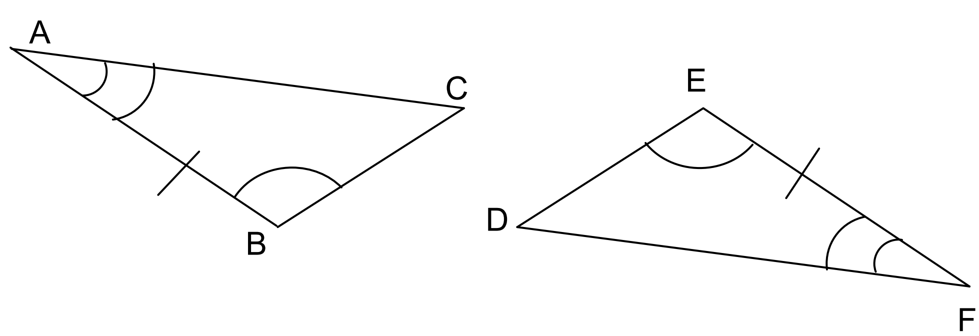
SSS Postulate: If the sides of one triangle are congruent to the sides of another triangle, the triangles are congruent.



SAS Postulate: If two sides and the included angle of one triangle are congruent to two sides and included angle of another triangles, the triangles are congruent.



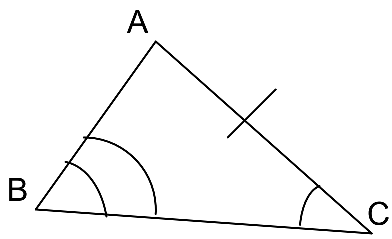
ASA Postulate: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent.

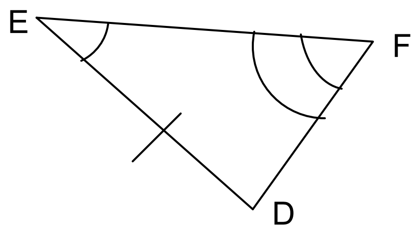


Congruent Triangle Theorems (pp. 2 of 5) KEY

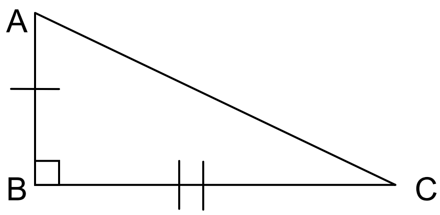
From the three postulates, four other methods arise.

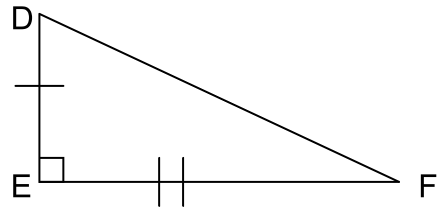
AAS Theorem: If two angles and the non-included side of one triangle are congruent to the two angles and non-included side of another triangle, the triangles are congruent.



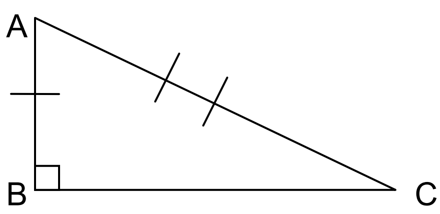


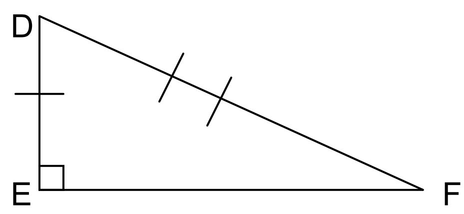
LL Theorem: In a right triangle, if the legs of one triangle are congruent to the corresponding legs of another triangle, the triangles are congruent.





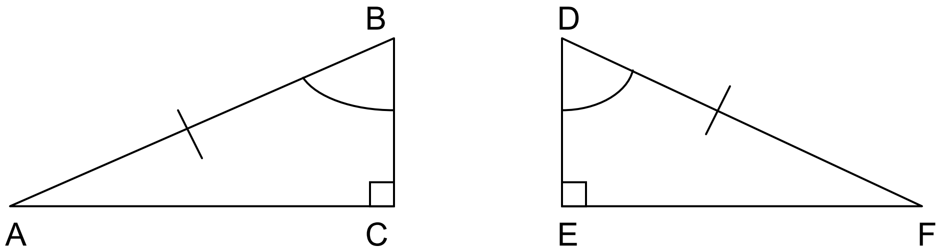
HL Theorem: In a right triangle, if the hypotenuse and leg of one triangle are congruent to the hypotenuse and corresponding leg of another triangle, the triangles are congruent.





Congruent Triangle Theorems (pp. 3 of 5) KEY

HA Theorem: In a right triangle, if the hypotenuse and one acute angle of the triangle are congruent to the hypotenuse and corresponding acute angle of another triangle, the triangles are congruent.

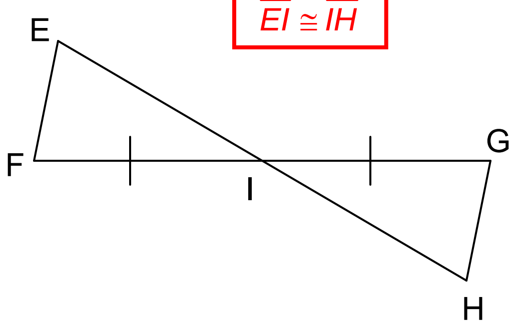


Examples:

1. Give any additional information that would be needed to prove the triangles congruent by the method given.

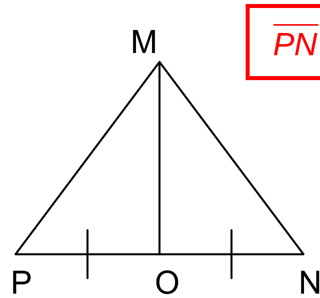
a. SAS

$\overline{EI} \cong \overline{IH}$



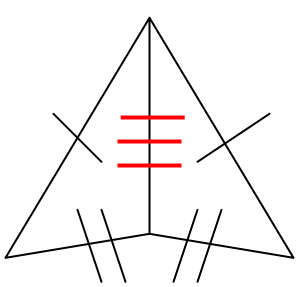
b. LL

$\overline{PN} \perp \overline{MO}$



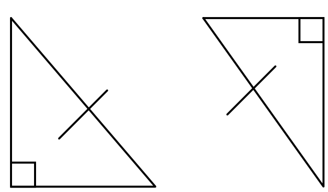
2. Determine which method can be used to prove the triangles congruent from the information given. For some pairs, it may not be possible to prove the triangles congruent. For these, explain what other information would be needed to prove congruence.

a.



SSS

b.

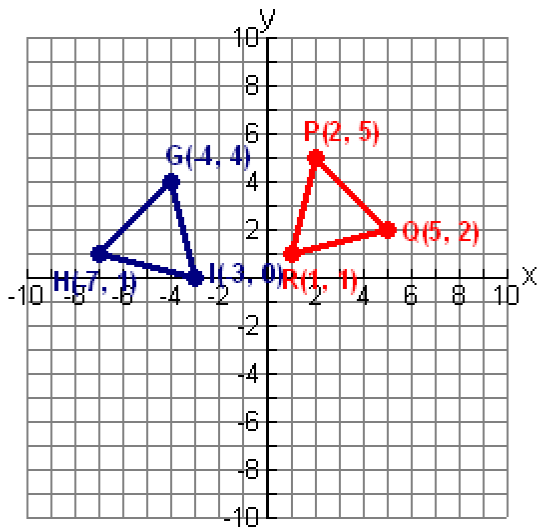


In order to prove congruence by HL, it is necessary to know that one of the corresponding legs is congruent.

Congruent Triangle Theorems (pp. 4 of 5) KEY

3. Plot each triangle in the coordinate plane. Find the lengths of each side. Use these values to determine if the triangles are congruent. Justify you answer.

   a. ΔPQR has vertices P(2,5), Q(5,2), R(1,1) and ΔGHI has vertices G(-4,4), H(-7,1), I(-3,0).



HI = 4.123
GI = 4.123
GH = 4.243

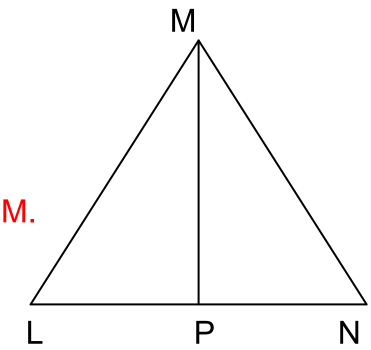
RQ = 4.123
RP = 4.123
PQ = 4.243

$\Delta GHI \cong \Delta PQR$ by SSS

Congruent Triangle Theorems (pp. 5 of 5) KEY

4. Use a flow chart proof to justify triangles congruent.

Given: ΔLMN is an isosceles triangle with vertex M.
$\overline{MP}$ is an altitude of ΔLMN.
Prove: $\Delta LMP \cong \Delta NMP$



Student samples will vary…
ΔLMN is an isosceles triangle with vertex M implies LM=NM.
$\overline{MP}$ is an altitude of ΔLMN implies right angles at point P,
Therefore ΔLMP and ΔNMP are right triangles.
$\overline{MP}$ is congruent to $\overline{MP}$ by reflexive property.
Therefore, $\Delta LMP \cong \Delta NMP$ by Hypotenuse Leg.

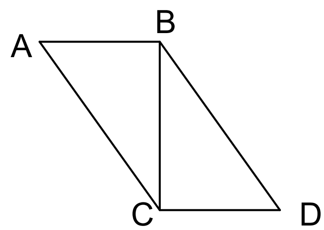
ΔLMN is an isosceles triangle with vertex M → $\overline{ML} \cong \overline{MN}$ → $\Delta LMP \cong \Delta NMP$ by SSS

$\overline{MP}$ is an altitude of ΔLMN → $\overline{MP}$ bisects $\overline{LN}$ → $\overline{LP} \cong \overline{PN}$ → $\Delta LMP \cong \Delta NMP$ by SSS

$\overline{MP} \cong \overline{MP}$ → $\Delta LMP \cong \Delta NMP$ by SSS

5. Use a two-column proof to justify triangles congruent.
Two possible samples are given below.

Given: $\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{BC}$
$\angle A \cong \angle D$
Prove: $\Delta ABC \cong \Delta DCB$



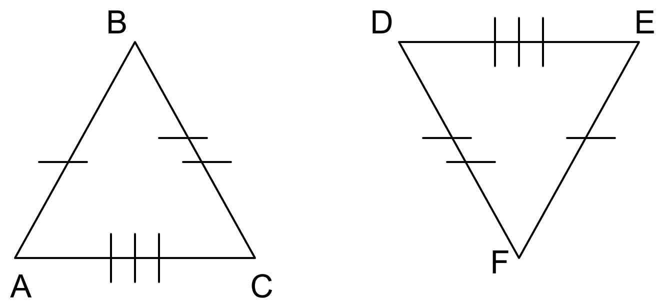
Statements	Reasons
$\overline{AB} \perp \overline{BC}$ $\overline{CD} \perp \overline{BC}$ $\angle A \cong \angle D$	Given
$m\angle ABC = 90$ $m\angle DCB = 90$	Definition of perpendicular lines
$m\angle ABC = m\angle DCB$	Transitive property
$\angle ABC \cong \angle DCB$	Definition of congruence
$\overline{BC} \cong \overline{BC}$	Reflexive property
$\Delta ABC \cong \Delta DCB$	AAS

Statements	Reasons
$\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{BC}$ $\angle A \cong \angle D$	Given
$m\angle ABC = 90^0$ and $m\angle BCD = 90^0$	Definition of perpendicular lines
ΔABC and ΔDCB are right triangles.	Definition of right triangles
$\overline{BC} \cong \overline{BC}$	Reflexive property
$\Delta ABC \cong \Delta DCB$	Leg Angle theorem

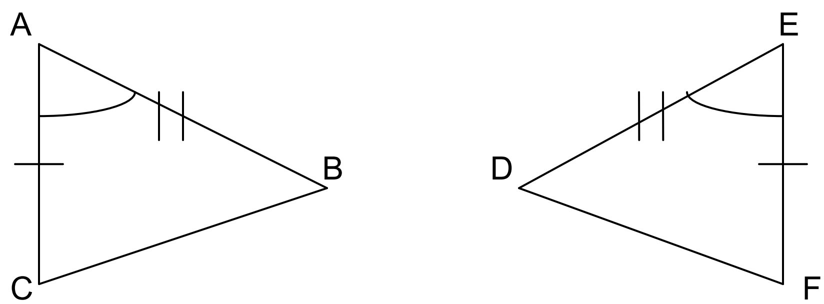
Congruent Triangle Theorems (pp. 1 of 5)

The following are seven methods by which to prove triangles are congruent, without having to find all corresponding parts congruent: SSS, SAS, ASA, AAS, and for right triangles only LL, HA, and HL.

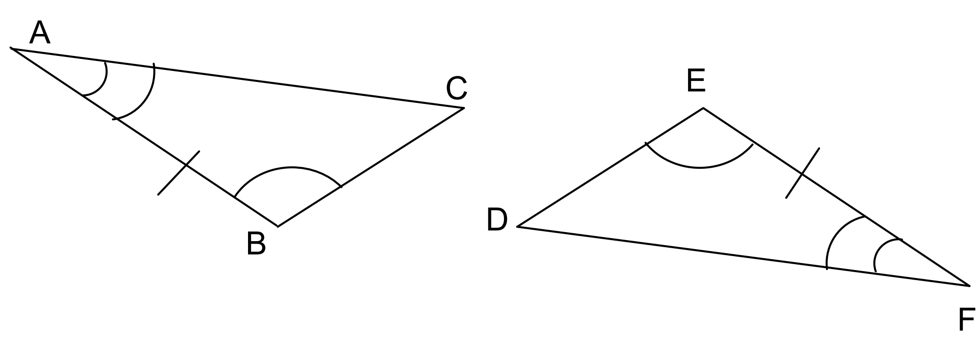
SSS Postulate: If the sides of one triangle are congruent to the sides of another triangle, the triangles are congruent.



SAS Postulate: If two sides and the included angle of one triangle are congruent to two sides and included angle of another triangles, the triangles are congruent.



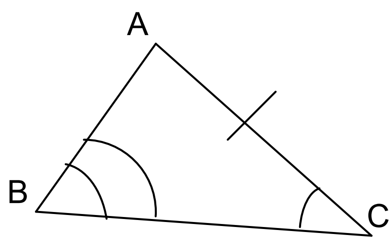
ASA Postulate: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent.

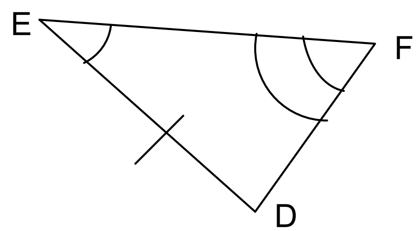


Congruent Triangle Theorems (pp. 2 of 5)

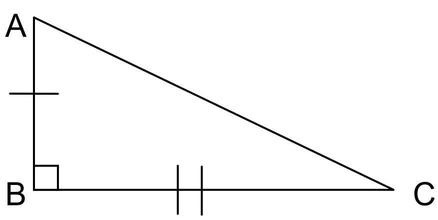
From the three postulates, four other methods arise.

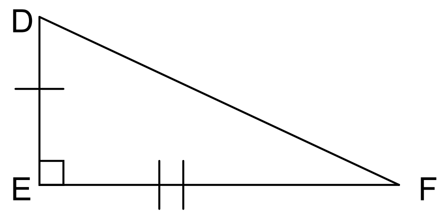
AAS Theorem: If two angles and the non-included side of one triangle are congruent to the two angles and non-included side of another triangle, the triangles are congruent.



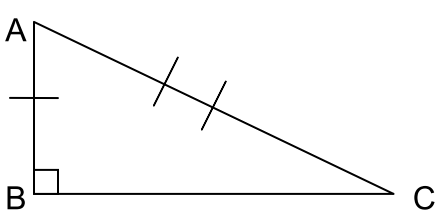


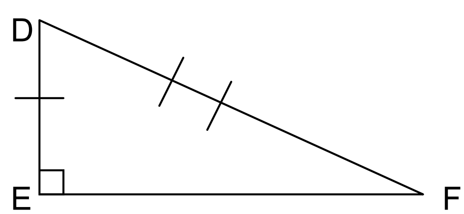
LL Theorem: In a right triangle, if the legs of one triangle are congruent to the corresponding legs of another triangle, the triangles are congruent.





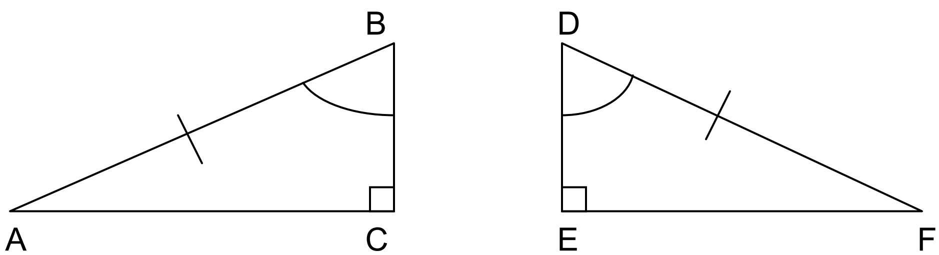
HL Theorem: In a right triangle, if the hypotenuse and leg of one triangle are congruent to the hypotenuse and corresponding leg of another triangle, the triangles are congruent.





Congruent Triangle Theorems (pp. 3 of 5)

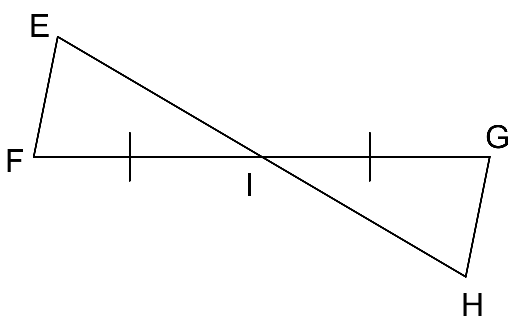
HA Theorem: In a right triangle, if the hypotenuse and one acute angle of the triangle are congruent to the hypotenuse and corresponding acute angle of another triangle, the triangles are congruent.



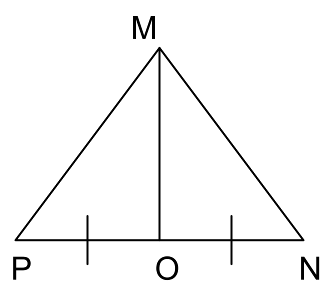
Examples:

1. Give any additional information that would be needed to prove the triangles congruent by the method given.

a. SAS

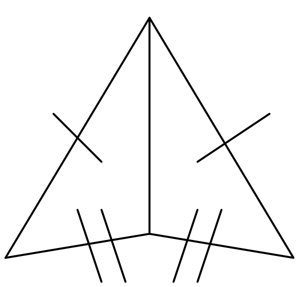


b. LL

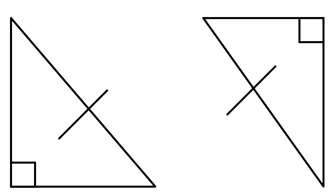


2. Determine which method can be used to prove the triangles congruent from the information given. For some pairs, it may be not possible to prove the triangles congruent. For these, explain what other information would be needed to prove congruence.

a.



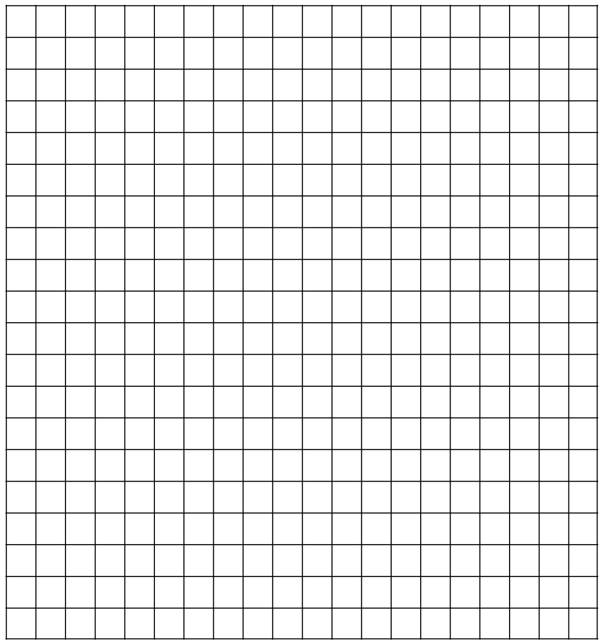
b.



Congruent Triangle Theorems (pp. 4 of 5)

3. Plot each triangle in the coordinate plane. Find the lengths of each side. Use these values to determine if the triangles are congruent. Justify you answer.

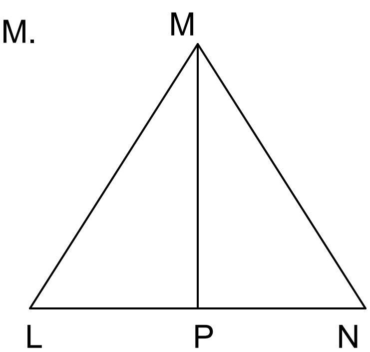
   a. ΔPQR has vertices P(2,5), Q(5,2), R(1,1) and ΔGHI has vertices G(-4,4), H(-7,1), I(-3,0).



Congruent Triangle Theorems (pp. 5 of 5)

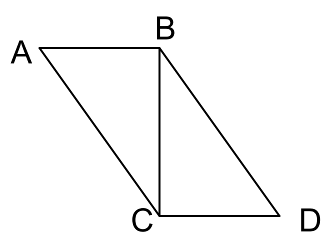
4. Use a flow chart proof to justify triangles congruent.

Given: ΔLMN is an isosceles triangle with vertex M.
$\overline{MP}$ is an altitude of ΔLMN.
Prove: $\Delta LMP \cong \Delta NMP$



5. Use a two-column proof to justify triangles congruent.

Given: $\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{BC}$
$\angle A \cong \angle D$
Prove: $\Delta ABC \cong \Delta DCB$



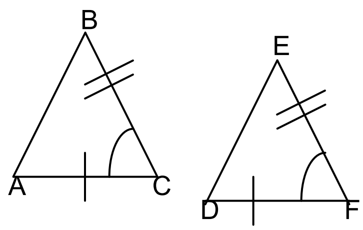
Statements	Reasons

What Makes Triangles Congruent? KEY

Study the two triangles for each problem. Circle the method that proves the triangles are congruent.

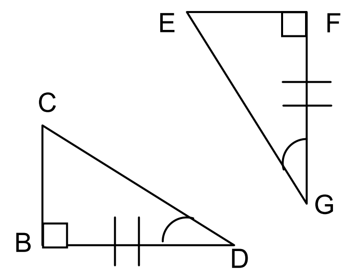
1. $\Delta ABC \cong \Delta DEF$

a. SSS
b. SAS
c. ASA



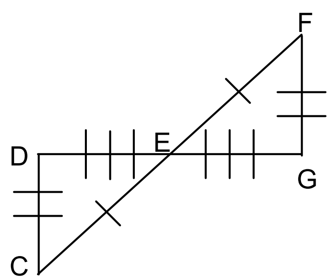
2. $\Delta BCD \cong \Delta FEG$

a. SSS
b. SAS
c. ASA



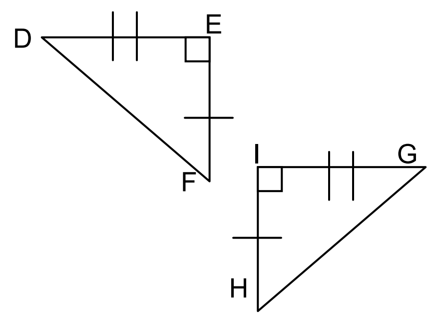
3. $\Delta CDE \cong \Delta FGE$

a. SSS or
b. SAS
c. ASA



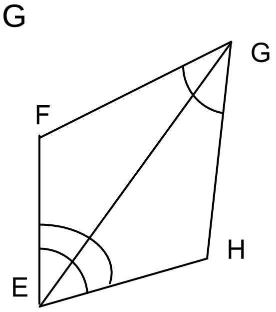
4. $\Delta DEF \cong \Delta HIG$

a. SSS
b. SAS
c. ASA



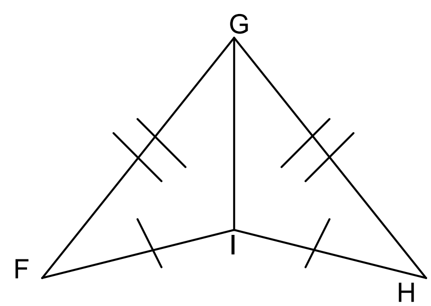
5. $\Delta EFG \cong \Delta EHG$

a. SSS
b. SAS
c. ASA



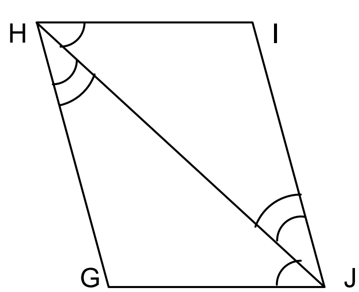
6. $\Delta FGI \cong \Delta HGI$

a. SSS
b. SAS
c. ASA



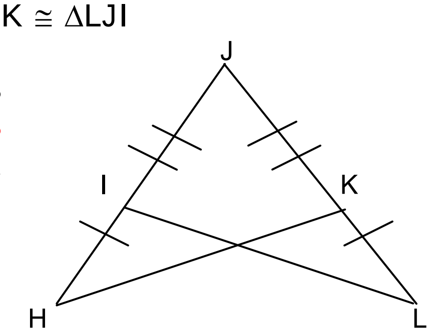
7. $\Delta GHJ \cong \Delta IJH$

a. SSS
b. SAS
c. ASA



8. $\Delta HJK \cong \Delta LJI$

a. SSS
b. SAS
c. ASA

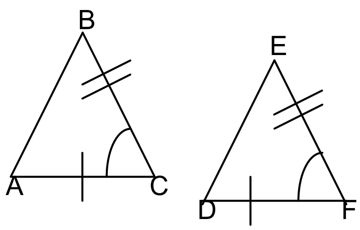


What Makes Triangles Congruent?

Study the two triangles for each problem. Circle the method that proves the triangles are congruent.

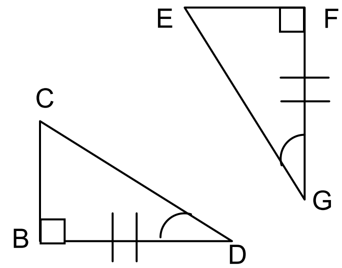
1. $\Delta ABC \cong \Delta DEF$

a. SSS
b. SAS
c. ASA



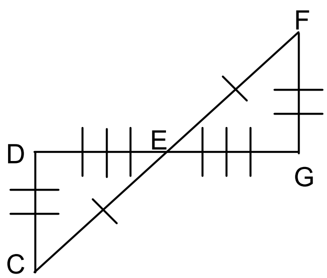
2. $\Delta BCD \cong \Delta FEG$

a. SSS
b. SAS
c. ASA



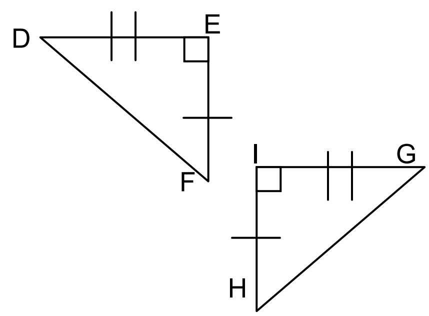
3. $\Delta CDE \cong \Delta FGE$

a. SSS
b. SAS
c. ASA



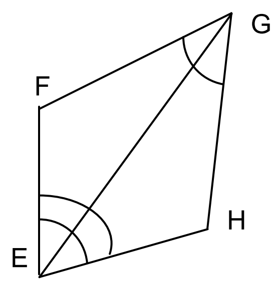
4. $\Delta DEF \cong \Delta HIG$

a. SSS
b. SAS
c.ASA



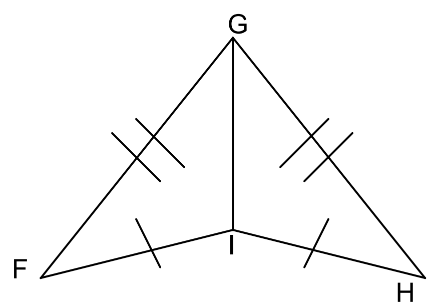
5. $\Delta EFG \cong \Delta EHG$

a. SSS
b. SAS
c. ASA



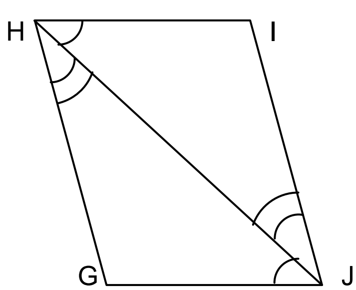
6. $\Delta FGI \cong \Delta HGI$

a. SSS
b. SAS
c. ASA



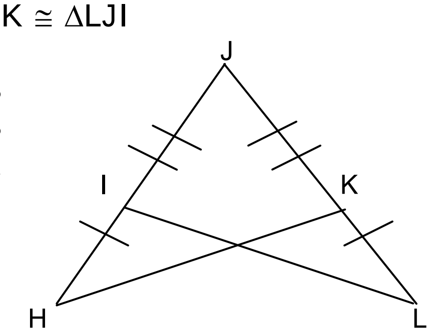
7. $\Delta GHJ \cong \Delta IJH$

a. SSS
b. SAS
c. ASA



8. $\Delta HJK \cong \Delta LJI$

a. SSS
b. SAS
c. ASA



Proving Triangles Congruent and CPCTC (pp. 1 of 3) KEY

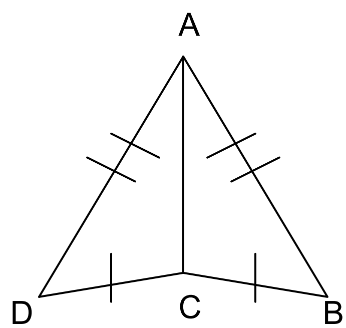
The definition of congruent triangles states two triangles are congruent *if and only if* their corresponding parts are congruent. *If and only if* is used when both the conditional and its converse are true. Therefore the converse is true:

Corresponding parts of congruent triangles are congruent. (CPCTC)

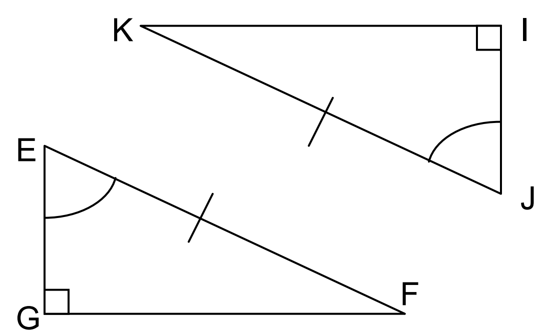
This can be used to prove parts of triangles congruent by first proving the triangles congruent.

Examples: Justify the following using two column or flow proofs.

1. Prove: $\angle D \cong \angle B$



2. Prove: $\overline{EG} \cong \overline{JI}$



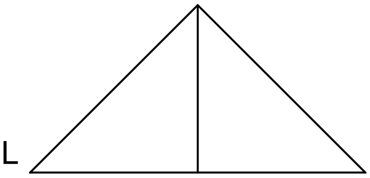
Teacher Notes:

1. Show triangles congruent by SSS and $\angle D \cong \angle B$ by CPCTC.
2. Show triangles congruent by AAS or HA and $\overline{EG} \cong \overline{JI}$ by CPCTC.

Proving Triangles Congruent and CPCTC (pp. 2 of 3) KEY

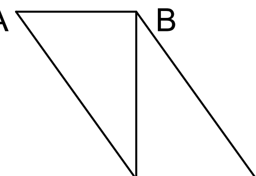
Practice Problems

1. Given: ΔLMN is an isosceles triangle with vertex M.
$\overline{MP}$ bisects $\overline{LN}$.
Prove: $\angle LMP \cong \angle NMP$



Statements	Reasons
ΔLMN is an isosceles triangle with vertex M. $\overline{MP}$ bisects $\overline{LN}$.	Given
$\overline{LM} \cong \overline{NM}$	Definition of Isosceles.
$\overline{LP} \cong \overline{NP}$	Definition of segment bisector.
$\overline{MP} \cong \overline{MP}$	Reflexive property
$\Delta LMP \cong \Delta NMP$	SSS
$\angle LMP \cong \angle NMP$	CPCTC

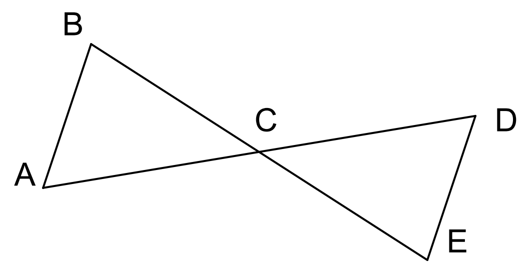
2. Given: $\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{BC}$
$\angle A \cong \angle D$
Prove: $\overline{AC} \cong \overline{DB}$



Statements	Reasons
$\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{BC}$ $\angle A \cong \angle D$	Given
$\angle ABC$ and $\angle DCB$ are right angles.	Definition of perpendicular lines.
$\overline{BC} \cong \overline{BC}$	Reflexive property.
$\Delta ABC \cong \Delta DCB$	Leg Angle
$\overline{AC} \cong \overline{DB}$	CPCTC

Proving Triangles Congruent and CPCTC (pp. 3 of 3) KEY

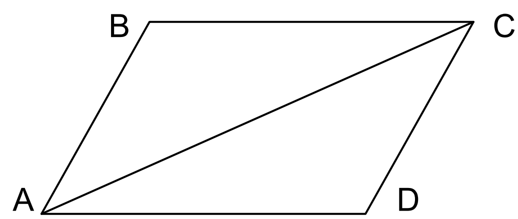
3. Given: C is the midpoint of $\overline{AD}$ and $\overline{BE}$
Prove: $\angle A \cong \angle D$



Statements	Reasons
C is the midpoint of $\overline{AD}$ and $\overline{BE}$	Given
$\overline{BC} \cong \overline{EC}$	Definition of midpoint
$\overline{AC} \cong \overline{DC}$	Definition of midpoint
$\angle ACB \cong \angle DCE$	Vertical angle theorem
$\Delta ABC \cong \Delta DEC$	SAS
$\angle A \cong \angle D$	CPCTC

4. Given: $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$

Prove: $\overline{AD} \cong \overline{CB}$



Statements	Reasons
$\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$	Given
$\angle BAC \cong \angle DCA$	Alt. Int. Angle Theorem
$\overline{AC} \cong \overline{AC}$	Reflexive
$\Delta ABC \cong \Delta CDA$	SAS
$\overline{AD} \cong \overline{CB}$	CPCTC

Proving Triangles Congruent and CPCTC (pp. 1 of 3)

The definition of congruent triangles states two triangles are congruent *if and only if* their corresponding parts are congruent. *If and only if* is used when both the conditional and its converse are true. Therefore the converse is true:

Corresponding parts of congruent triangles are congruent. (CPCTC)

This can be used to prove parts of triangles congruent by first proving the triangles congruent.

Examples: Justify the following using two column or flow proofs.

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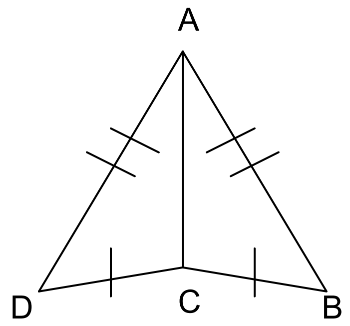
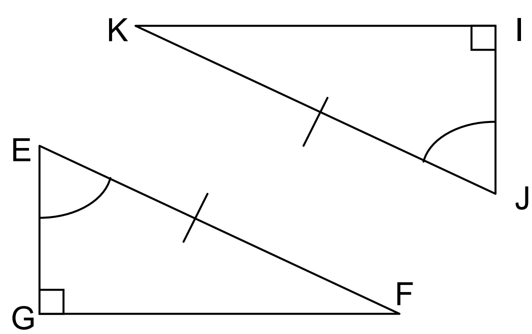


2. Prove: $\overline{EG} \cong \overline{JI}$



Proving Triangles Congruent and CPCTC (pp. 2 of 3)

Practice Problems

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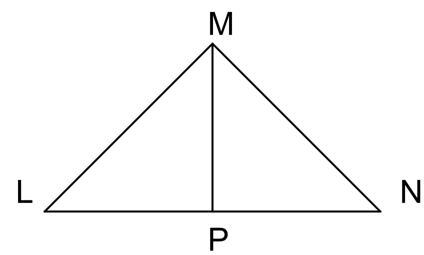
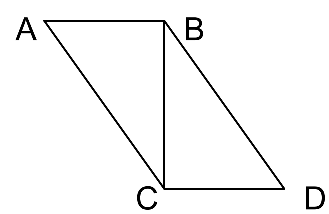
$\overline{MP}$ bisects $\overline{LN}$.
Prove: $\angle LMP \cong \angle NMP$

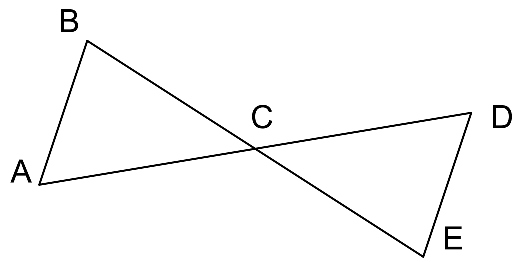


2. Given: $\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{BC}$
$\angle A \cong \angle D$
Prove: $\overline{AC} \cong \overline{DB}$



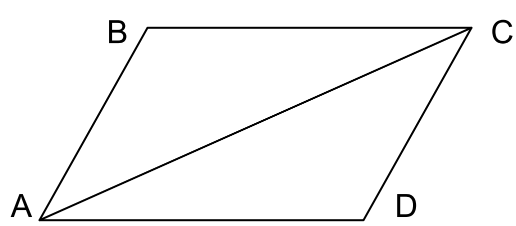
Proving Triangles Congruent and CPCTC (pp. 3 of 3)

3. Given: C is the midpoint of $\overline{AD}$ and $\overline{BE}$
   Prove: $\angle A \cong \angle D$



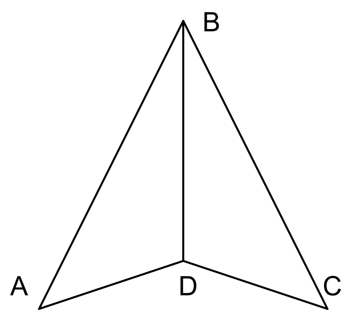
4. Given: $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$

   Prove: $\overline{AD} \cong \overline{CB}$



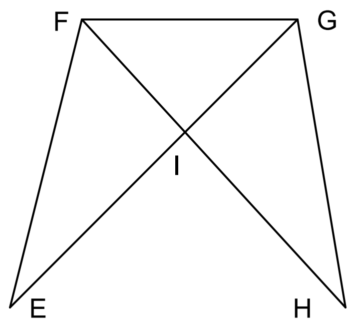
Sneaky Triangles KEY

Study the figure on the left. Find a pair of corresponding parts from the numbered and lettered columns on the right. Put the letter in the box below that corresponds to the appropriate number.



$\Delta ABD \cong \Delta CBD$

1. $\angle A$	S. $\overline{BD}$
2. $\angle ABD$	I. $\angle CDB$
3. $\angle BDA$	E. $\angle C$
4. $\overline{AB}$	A. $\overline{CD}$
5. $\overline{BD}$	E. $\overline{BC}$
6. $\overline{AD}$	O. $\angle DBC$

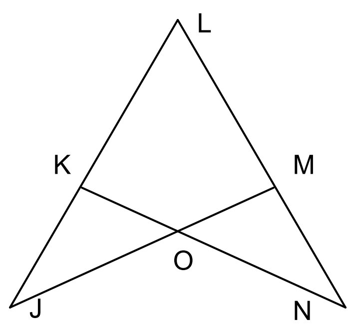


$\Delta EFG \cong \Delta HGF$

7. $\angle E$	R. $\overline{FH}$
8. $\angle EFG$	O. $\angle H$
9. $\angle FGE$	E. $\angle FGH$
10. $\overline{EF}$	E. $\overline{FG}$
11. $\overline{FG}$	I. $\angle GFH$
12. $\overline{GE}$	S. $\overline{GH}$

$\Delta EFI \cong \Delta HGI$

13. $\angle E$	S. $\angle HGI$
14. $\angle EFI$	C. $\overline{IH}$
15. $\angle FIE$	U. $\angle GIH$
16. $\overline{FE}$	E. $\overline{GH}$
17. $\overline{FI}$	E. $\angle H$
18. $\overline{IE}$	R. $\overline{GI}$



$\Delta JLM \cong \Delta NLK$

19. $\angle J$	T. $\angle LKN$
20. $\angle L$	R. $\angle L$
21. $\angle LMJ$	T. $\overline{KN}$
22. $\overline{LJ}$	V. $\overline{LN}$
23. $\overline{LM}$	D. $\overline{LK}$
24. $\overline{MJ}$	E. $\angle N$

$\Delta JKO \cong \Delta NMO$

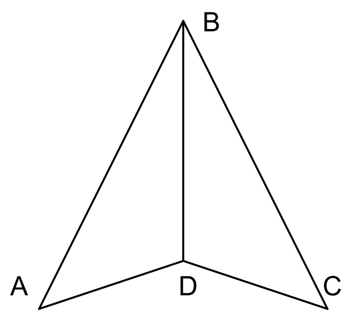
25. $\angle J$	C. $\overline{ON}$
26. $\angle JKO$	P. $\overline{MO}$
27. $\angle KOJ$	S. $\angle NMO$
28. $\overline{KJ}$	N. $\angle MON$
29. $\overline{KO}$	T. $\angle N$
30. $\overline{OJ}$	T. $\overline{MN}$

5	25	7	17	8	23	13	28	1	18	24	3	22	11	14
S	T	O	R	E	D	E	T	E	C	T	I	V	E	S

6	20	16	30	2	15	27	21	4	12	26	29	9	19	10
A	R	E	C	O	U	N	T	E	R	S	P	I	E	S

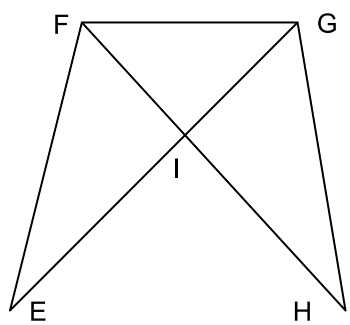
Sneaky Triangles

Study the figure on the left. Find a pair of corresponding parts from the numbered and lettered columns on the right. Put the letter in the box below that corresponds to the appropriate number.



$\triangle ABD \cong \triangle CBD$

1. $\angle A$ — S. $\overline{BD}$
2. $\angle ABD$ — I. $\angle CDB$
3. $\angle BDA$ — E. $\angle C$
4. $\overline{AB}$ — A. $\overline{CD}$
5. $\overline{BD}$ — E. $\overline{BC}$
6. $\overline{AD}$ — O. $\angle DBC$

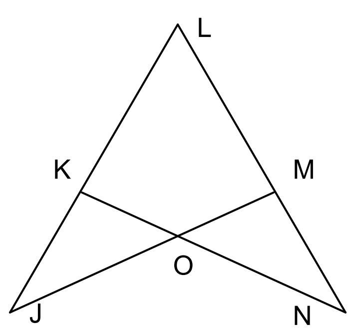


$\triangle EFG \cong \triangle HGF$

7. $\angle E$ — R. $\overline{FH}$
8. $\angle EFG$ — O. $\angle H$
9. $\angle FGE$ — E. $\angle FGH$
10. $\overline{EF}$ — E. $\overline{FG}$
11. $\overline{FG}$ — I. $\angle GFH$
12. $\overline{GE}$ — S. $\overline{GH}$

$\triangle EFI \cong \triangle HGI$

13. $\angle E$ — S. $\angle HGI$
14. $\angle EFI$ — C. $\overline{IH}$
15. $\angle FIE$ — U. $\angle GIH$
16. $\overline{FE}$ — E. $\overline{GH}$
17. $\overline{FI}$ — E. $\angle H$
18. $\overline{IE}$ — R. $\overline{GI}$



$\triangle JLM \cong \triangle NLK$

19. $\angle J$ — T. $\angle LKN$
20. $\angle L$ — R. $\angle L$
21. $\angle LMJ$ — T. $\overline{KN}$
22. $\overline{LJ}$ — V. $\overline{LN}$
23. $\overline{LM}$ — D. $\overline{LK}$
24. $\overline{MJ}$ — E. $\angle N$

$\triangle JKO \cong \triangle NMO$

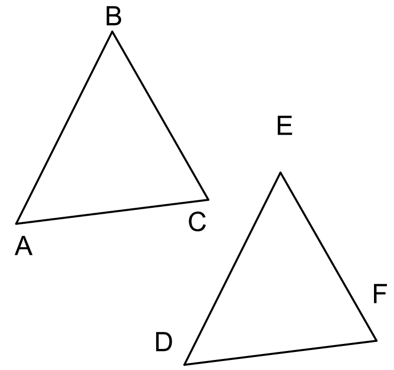
25. $\angle J$ — C. $\overline{ON}$
26. $\angle JKO$ — P. $\overline{MO}$
27. $\angle KOJ$ — S. $\angle NMO$
28. $\overline{KJ}$ — N. $\angle MON$
29. $\overline{KO}$ — T. $\angle N$
30. $\overline{OJ}$ — T. $\overline{MN}$

5	25	7	17	8	23	13	28	1	18	24	3	22	11	14

6	20	16	30	2	15	27	21	4	12	26	29	9	19	10

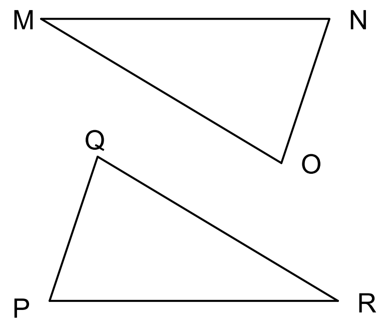
Tall Tom's Short Pants KEY

Tall Tom is buying new pants for school. He tried on the same size he wore last year. He decided he would need to buy longer pants. To find out how he knew, study the figures below. Use patty paper and transformations to determine how the triangles are congruent. Find a pair of corresponding parts from the numbered and lettered columns on the right. Put the letter in the box below that corresponds to the appropriate number.



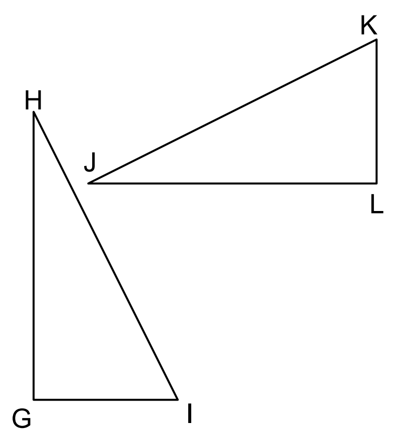
$\underline{\Delta ABC} \cong \underline{\Delta DEF}$

1. $\angle A$	O. $\overline{EF}$
2. $\angle B$	S. $\angle F$
3. $\angle C$	E. $\angle D$
4. $\overline{AB}$	S. $\overline{FD}$
5. $\overline{BC}$	C. $\angle E$
6. $\overline{CA}$	E. $\overline{DE}$



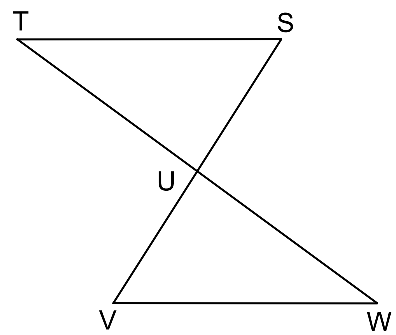
$\underline{\Delta OMN} \cong \underline{\Delta QRP}$

13. $\angle M$	O. $\angle P$
14. $\angle N$	H. $\overline{PQ}$
15. $\angle O$	U. $\overline{RP}$
16. $\overline{MN}$	F. $\angle R$
17. $\overline{NO}$	G. $\overline{QR}$
18. $\overline{OM}$	T. $\angle Q$



$\underline{\Delta HGI} \cong \underline{\Delta JLK}$

7. $\angle G$	T. $\overline{LJ}$
8. $\angle H$	I. $\overline{JK}$
9. $\angle I$	T. $\angle L$
10. $\overline{GH}$	S. $\overline{KL}$
11. $\overline{HI}$	O. $\angle J$
12. $\overline{IG}$	K. $\angle K$



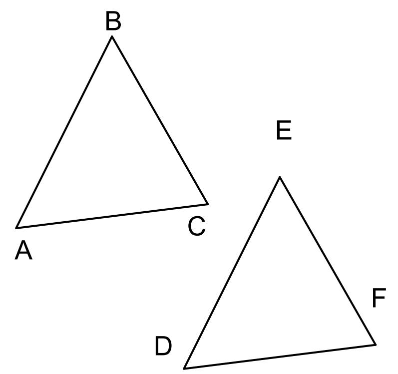
$\underline{\Delta TSU} \cong \underline{\Delta WVU}$

19. $\angle S$	L. $\overline{WU}$
20. $\angle T$	U. $\angle V$
21. $\angle TUS$	W. $\overline{VW}$
22. $\overline{ST}$	E. $\angle W$
23. $\overline{TU}$	T. $\angle WUV$
24. $\overline{US}$	F. $\overline{UV}$

7	22	14	24	1	20	15	8	13	17	11	6	23	4	18	3	12	21	16	2	9	5	19	10
T	W	O	F	E	E	T	O	F	H	I	S	L	E	G	S	S	T	U	C	K	O	U	T

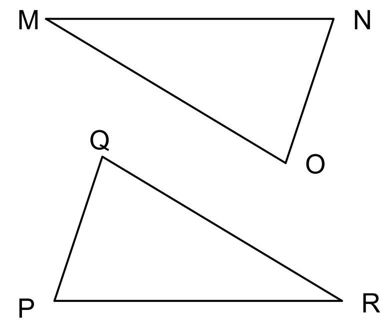
Tall Tom's Short Pants

Tall Tom is buying new pants for school. He tried on the same size he wore last year. He decided he would need to buy longer pants. To find out how he knew, study the figures below. Use patty paper and transformations to determine how the triangles are congruent. Find a pair of corresponding parts from the numbered and lettered columns on the right. Put the letter in the box below that corresponds to the appropriate number.



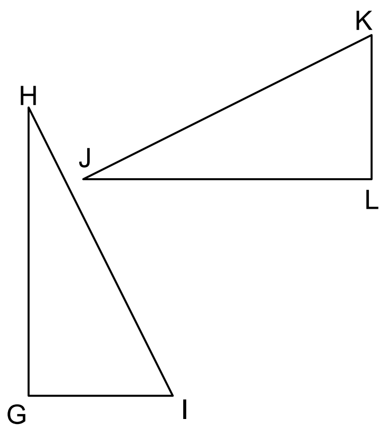
______ $\cong$ ______

1. $\angle A$	O. $\overline{EF}$
2. $\angle B$	S. $\angle F$
3. $\angle C$	E. $\angle D$
4. $\overline{AB}$	S. $\overline{FD}$
5. $\overline{BC}$	C. $\angle E$
6. $\overline{CA}$	E. $\overline{DE}$



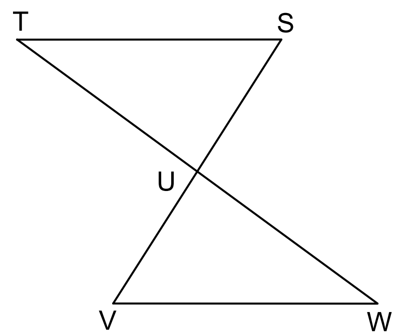
______ $\cong$ ______

13. $\angle M$	O. $\angle P$
14. $\angle N$	H. $\overline{PQ}$
15. $\angle O$	U. $\overline{RP}$
16. $\overline{MN}$	F. $\angle R$
17. $\overline{NO}$	G. $\overline{QR}$
18. $\overline{OM}$	T. $\angle Q$



______ $\cong$ ______

7. $\angle G$	T. $\overline{LJ}$
8. $\angle H$	I. $\overline{JK}$
9. $\angle I$	T. $\angle L$
10. $\overline{GH}$	S. $\overline{KL}$
11. $\overline{HI}$	O. $\angle J$
12. $\overline{IG}$	K. $\angle K$



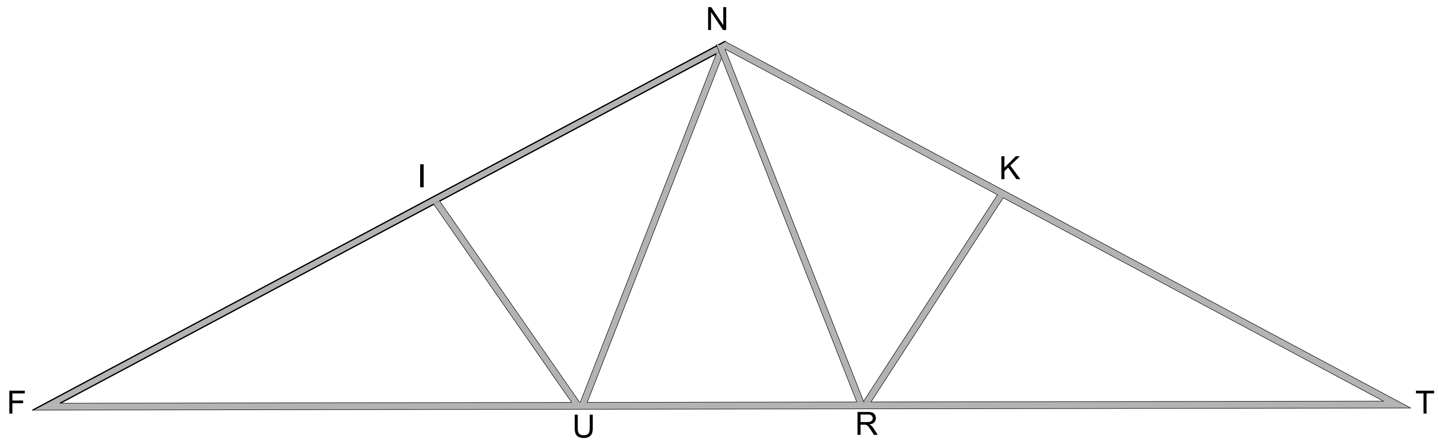
______ $\cong$ ______

19. $\angle S$	L. $\overline{WU}$
20. $\angle T$	U. $\angle V$
21. $\angle TUS$	W. $\overline{VW}$
22. $\overline{ST}$	E. $\angle W$
23. $\overline{TU}$	T. $\angle WUV$
24. $\overline{US}$	F. $\overline{UV}$

7	22	14	24	1	20	15	8	13	17	11	6	23	4	18	3	12	21	16	2	9	5	19	10

Over the Roof KEY

In a previous lesson, you discovered classifications of triangles and triangle properties that can be used to find angle values and side lengths in an isosceles triangle. Use your previous knowledge and your understanding of congruent triangles to answer the following questions.



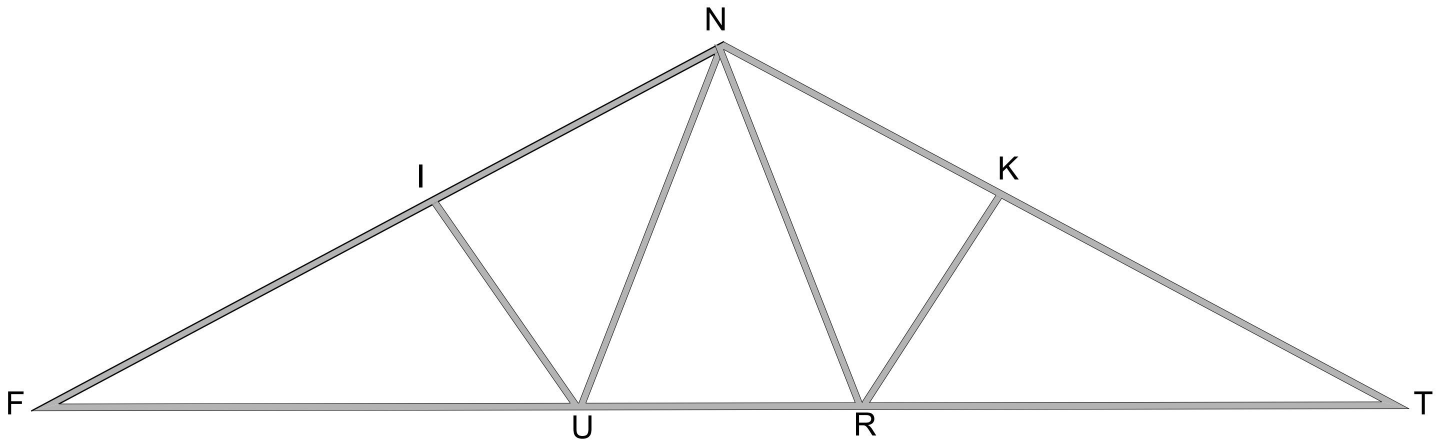
1. In the Fink truss pictured above,
   a. Which triangles appear to be congruent? ΔFUN and ΔTRN ; ΔFUI and ΔTRK ; ΔIUN and ΔKRN
   b. Explain how congruence transformations could be used to show triangles congruent in the picture above. A reflection over a vertical line passing through point N shows the above triangle pairs congruent.
2. Using one of the pairs of triangles from question 1a, explain what is meant by a congruence correspondence. A congruence correspondence pairs vertices from one triangle to those of another in such a way that the triangles are congruent. For example, $\Delta FUN \cong \Delta TRN$ is the same congruence correspondence as $\Delta FNU \cong \Delta TNR$, because in each statement the mapping of the triangles' vertices is the same.
3. Suppose ΔUNR is Isosceles such that $\angle N$ is the vertex angle, $FU = TR$ and $\angle FUN \cong \angle TRN$. Use this given information to prove (two column, paragraph, or flow proof) that $\Delta FUN \cong \Delta TRN$.
   Student answers will vary.
   Two column example below.

Statements	Reasons
ΔUNR is Isosceles such that $\angle N$ is the vertex angle and $FU = TR$ and $\angle FUN \cong \angle TRN$.	Given.
$\overline{NR} \cong \overline{NU}$	Definition of Isosceles.
$\Delta FUN \cong \Delta TRN$	SAS

4. Explain how your proof from question 3 can be used to prove other parts of the triangles congruent.
   After proving the triangles congruent, it follows that other corresponding parts such as $\overline{FN}$ and $\overline{TN}$ are congruent by CPCTC

Over the Roof

In a previous lesson, you discovered classifications of triangles and triangle properties that can be used to find angle values and side lengths in an isosceles triangle. Use your previous knowledge and your understanding of congruent triangles to answer the following questions.



1. In the Fink truss pictured above

    a. Which triangles appear to be congruent?

    b. Explain how congruence transformations could be used to show triangles congruent in the picture above.

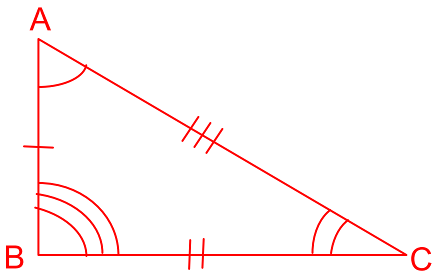
2. Using one of the pairs of triangles from question 1a, explain what is meant by a congruence correspondence.

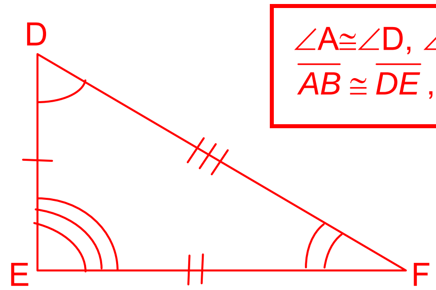
3. Suppose ΔUNR is Isosceles such that $\angle N$ is the vertex angle, $FU = TR$ and $\angle FUN \cong \angle TRN$. Use this given information to prove (two column, paragraph, or flow proof) that $\Delta FUN \cong \Delta TRN$.

4. Explain how your proof from question 3 can be used to prove other parts of the triangles congruent.

Evaluating Congruent Triangles (pp. 1 of 4) KEY

1. Given $\Delta ABC \cong \Delta DEF$. Draw representative triangles, marking the congruent parts. Identify the corresponding parts symbolically.

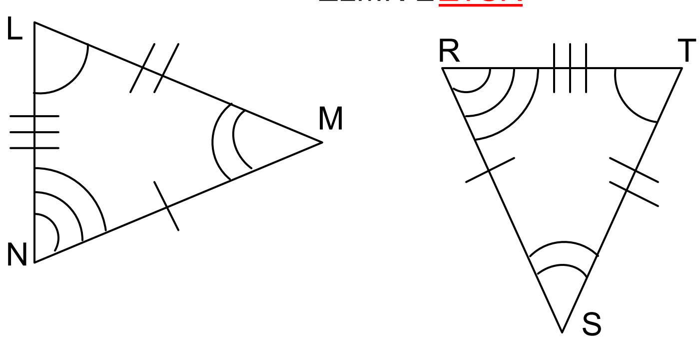




$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
$\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

2. Study the triangles below and complete the congruence statement.

$\Delta LMN \cong \Delta TSR$



3. Given $\Delta JKL \cong \Delta MNO$.
   a. $\overline{JK} \cong \overline{MN}$

   b. JK = 3x – 4 , MN = x + 6, find x, JK, and MN.
      3x – 4 = x + 6
      2x = 10
      x = 5
      JK = 11
      MN = 11

   c. $\angle O \cong \angle L$

   d. $m\angle O = (2d + 8)^{\circ}$, $m\angle L = (7d - 12)^{\circ}$, find d, $\angle O$, and $\angle L$.
      2d + 8 = 7d – 12
      20 = 5d
      d = 4
      $\angle O = 16^{\circ}$
      $\angle L = 16^{\circ}$

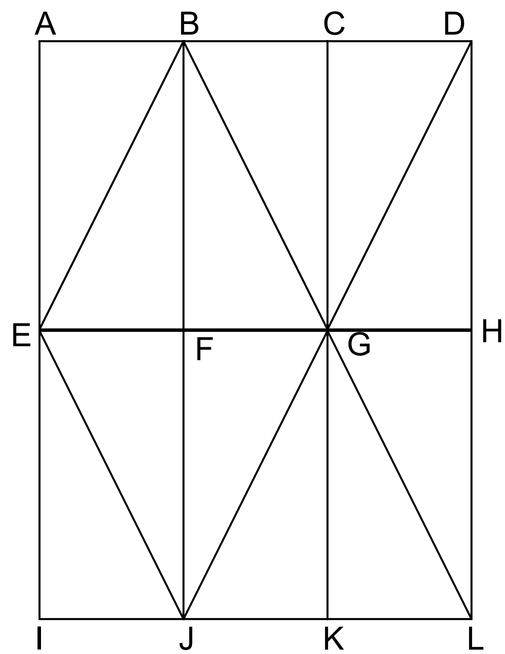
Evaluating Congruent Triangles (pp. 2 of 4) KEY

4. Look at the figure below. Identify one set of triangles that illustrate each of the following congruence transformations.

   a. Translation
   Answers will vary.
   Sample:
   $\triangle ABE \cong \triangle CDG$

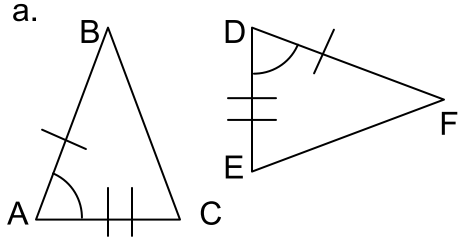
   b. Rotation
   Answers will vary.
   Sample:
   $\triangle BFG \cong \triangle JFE$

   c. Reflection
   Answers will vary.
   Sample:
   $\triangle EBF \cong \triangle GBF$



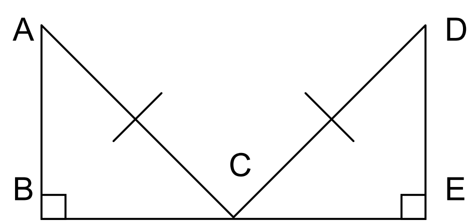
5. Determine which method can be used to prove the triangles congruent from the information given. If there is not enough given, explain what extra information would be needed to prove congruence.

   a.



$\triangle ABC \cong \triangle DFC$ by SAS.

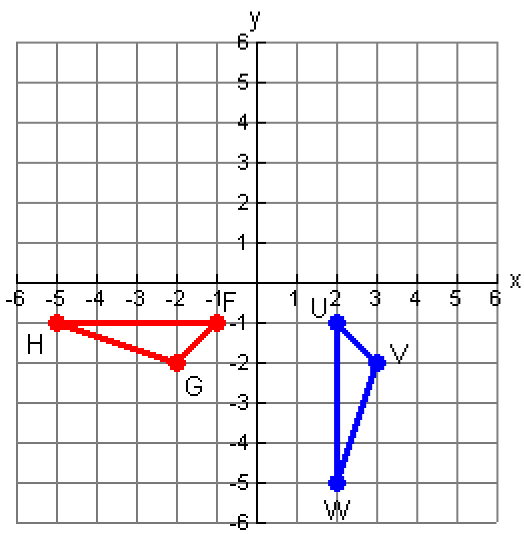
   b.



Not enough information is given. In order to prove congruency by HL, you would need to know that one corresponding set of legs is congruent.

Evaluating Congruent Triangles (pp. 3 of 4) KEY

6. Plot each triangle on the coordinate plane. Find the length of each side. Use these values to determine if the triangles are congruent. Justify your reasoning.
   ΔFGH has vertices F(-1,-1), G(-2,-2), H(-5,-1)
   ΔUVW has vertices U(2,-1), V(3,-2), W(2,-5).



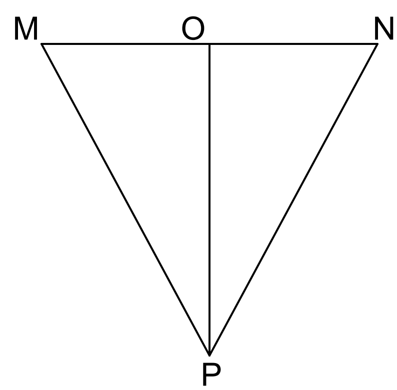
FG = $\sqrt{2} = 1.4$
GH = $\sqrt{10} = 3.16$
FH = 4

UV = $\sqrt{2} = 1.4$
VW = $\sqrt{10} = 3.16$
UW = 4

$\Delta FGH \cong \Delta UVW$ by SSS.

7. Justify the following with a flow chart proof.
   Given: $\overline{OP}$ is the perpendicular bisector of $\overline{MN}$
   Prove: $\Delta MOP \cong \Delta NOP$

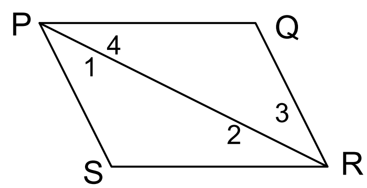
$\overline{OP}$ is the perpendicular bisector of $\overline{MN}$, therefore $\angle MOP$ is a right angle and $\overline{MO} \cong \overline{ON}$ by definition of perpendicular bisector. $\overline{OP} \cong \overline{OP}$ by the reflexive property. $\Delta MOP \cong \Delta NOP$ by the LL theorem.



Evaluating Congruent Triangles (pp. 4 of 4) KEY

8. Justify the following with a two-column proof.
   Given: $\overline{SR} \parallel \overline{PQ}$; $\overline{SP} \parallel \overline{QR}$

   Prove: $\Delta SPR \cong \Delta QRP$

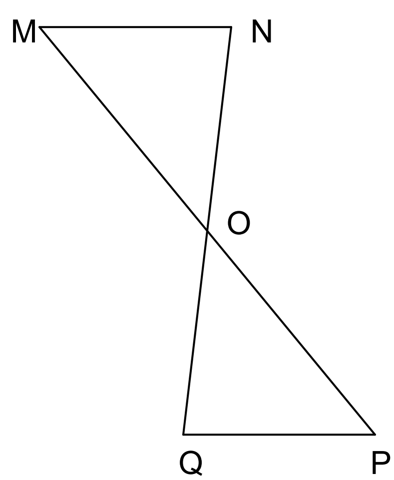


Statements	Reasons
1. $\overline{SR} \parallel \overline{PQ}$; $\overline{SP} \parallel \overline{QR}$	1. Given
2. $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$	2. If lines are parallel, alternate interior angles are congruent.
3. $\overline{PR} \cong \overline{PR}$	3. Reflexive property
4. $\Delta SPR \cong \Delta QRP$	4. ASA

9. Justify the following by a method of choice.
   Given: $\overline{NO} \cong \overline{QO}$, $\angle N \cong \angle Q$
   Prove: $\angle P \cong \angle M$

Answers will vary. Sample:

Statements	Reasons
$\overline{NO} \cong \overline{QO}$, $\angle N \cong \angle Q$	Given
$\angle NOM \cong \angle QOP$	Vertical angles are congruent.
$\Delta NOM \cong \Delta QOP$	ASA
$\angle P \cong \angle M$	CPCTC

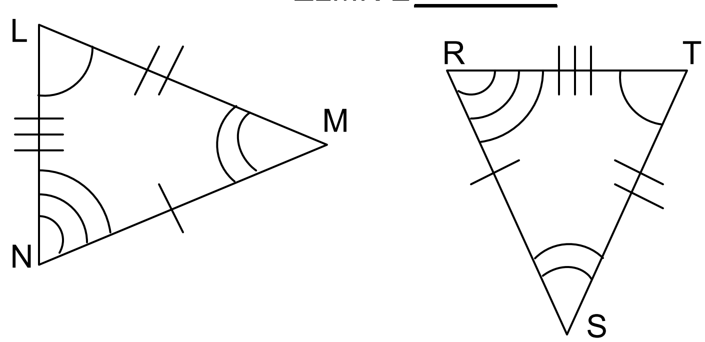


Evaluating Congruent Triangles (pp. 1 of 4)

1. Given $\Delta ABC \cong \Delta DEF$. Draw representative triangles, marking the congruent parts. Identify the corresponding parts symbolically.

2. Study the triangles below and complete the congruence statement.

$\Delta LMN \cong$ ________



3. Given $\Delta JKL \cong \Delta MNO$.
   a. $\overline{JK} \cong$ ________
   b. JK = 3x – 4 , MN = x + 6, find x, JK, and MN.
   c. $\angle O \cong$ ________
   d. $m\angle O = (2d + 8)^o$, $m\angle L = (7d - 12)^o$, find d, $\angle O$, and $\angle L$.

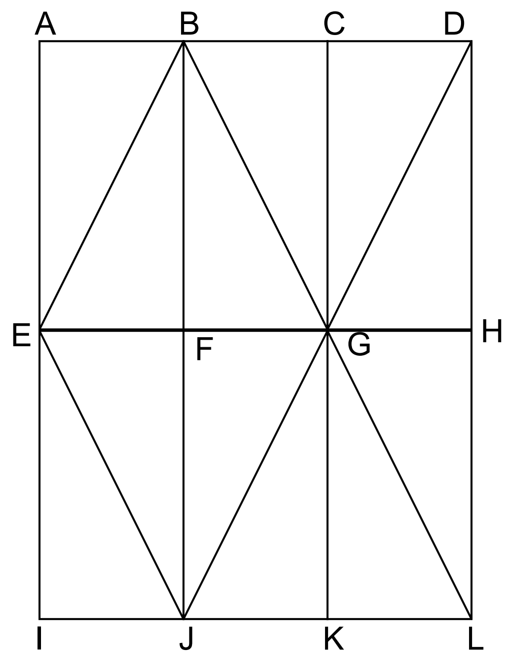
Evaluating Congruent Triangles (pp. 2 of 4)

4. Look at the figure below. Identify one set of triangles that illustrate each of the following congruence transformations.

   a. Translation

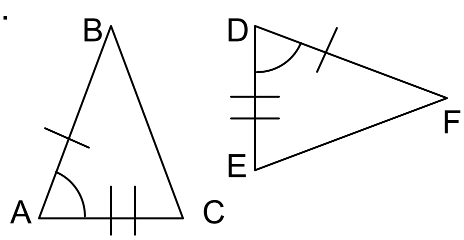
   b. Rotation

   c. Reflection

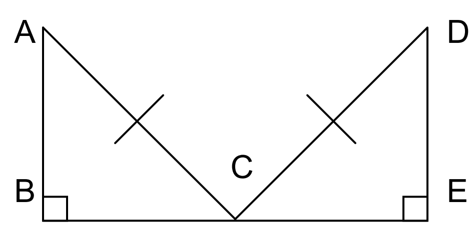


5. Determine which method can be used to prove the triangles congruent from the information given. If there is not enough given, explain what extra information would be needed to prove congruence.

a.

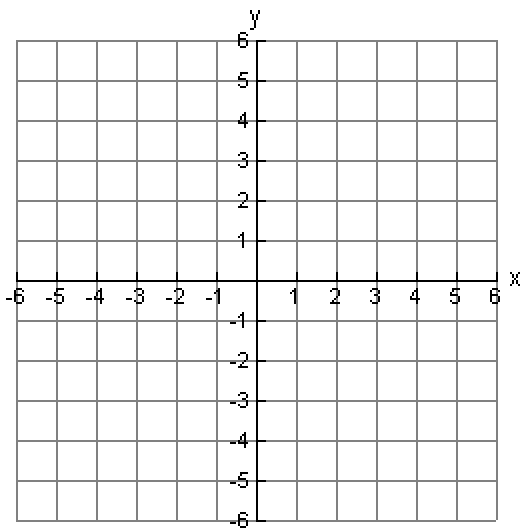


b.



Evaluating Congruent Triangles (pp. 3 of 4)

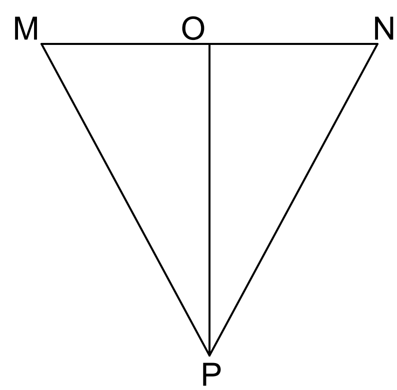
6. Plot each triangle on the coordinate plane. Find the length of each side. Use these values to determine if the triangles are congruent. Justify your reasoning.
   ΔFGH has vertices F(-1,-1), G(-2,-2), H(-5,-1)
   ΔUVW has vertices U(2,-1), V(3,-2), W(2,-5).



FG = ___________
GH = ___________
FH = ___________

UV = ___________
VW = ___________
UW = ___________

7. Justify the following with a flow chart proof.
   Given: $\overline{OP}$ is the perpendicular bisector of $\overline{MN}$
   Prove: $\Delta MOP \cong \Delta NOP$



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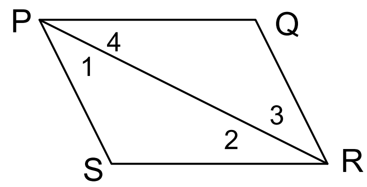
8. Justify the following with a two-column proof.
   Given: $\overline{SR} \,||\, \overline{PQ}$; $\overline{SP} \parallel \overline{QR}$
   Prove: $\Delta SPR \cong \Delta QRP$



Statements	Reasons
1. $\overline{SR} \,||\, \overline{PQ}$; $\overline{SP} \parallel \overline{QR}$	1. Given
2. $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$	2.
3. $\overline{PR} \cong \overline{PR}$	3.
4. $\Delta SPR \cong \Delta QRP$	4.

9. Justify the following by a method of choice.
   Given: $\overline{NO} \cong \overline{QO}$, $\angle N \cong \angle Q$
   Prove: $\angle P \cong \angle M$